the total possible errors and, if necessary, to go to one finer plot. However, this can be repeated to any given accuracy; in other words, it is sufficient to refine the plot if the difference between the bound and the correct value is to be made smaller than any given value. This shall now be shown briefly.

The error in (19) consists of two parts: 1) Error due to incomplete relaxation, 2) Inherent error due to size of the grid.

The first error is essentially numerical and shall not be of concern here. It is assumed that a sufficiently accurate relaxation is available. For the second error, however, we will assume that we know the exact solution ϕ and we will take arbitrarily the values of the exact solution as grid values U_{k1} etc., and get

$$U_k = \phi_k.$$

With (19) this error becomes

$$\iint (\nabla\phi')^2 dA.$$

But on the grid corners $\phi_k' = 0$; and because ϕ is continuous and differentiable the integral, and with it the error for this arbitrarily chosen plot, will disappear in the limit if the grid is made sufficiently fine.

Of course the plot for which it was just proved that its error can be made negligible is not the plot obtained by relaxation. However, it is a pleasant feature of the variational method as compared to conventional relaxation that this does not really matter because the error due to the completely relaxed plot must be still smaller than the error from any other plot. This completes the proof.

Ferrite Line Width Measurements in a Cross-Guide Coupler*

DONALD C. STINSON†

***Summary*—Theoretical and experimental results are presented to show that the line width and the *g* factor of a spherical ferrite sample can be measured in a cross-guide coupler. The method is much easier to instrument than the usual cavity method and the measurements are much easier to perform. Experimental verification with a cavity perturbation system indicates that the measured quantities are sufficiently accurate for most purposes.**

INTRODUCTION

RECENT work by the author with ferrite directional couplers indicated that such devices[1] might be useful in measuring the line width and the *g* factor of ferrites. Earlier unpublished work by the author in this field showed that such devices were not practical for measuring the components of the susceptibility tensor of a ferrite sphere because of the extreme sensitivity that was required in the detecting system. However, the measurement of the line width and the *g* factor depends upon relative power measurements. Thus, it was felt that the couplers offered considerable promise of yielding accurate data with a minimum of effort. A second method was also desirable in order to verify data on line widths and *g* factors obtained with a cavity-perturbation system. It was not intended that the new method be highly accurate, but rather that it be simple and capable of yielding reliable comparative data during the development of optimum manufacturing techniques.

The method chosen is extremely simple and uses a cross-guide directional coupler with a round, centered hole in the common broad wall. The wall thickness at the coupling aperture is about half the normal waveguide wall thickness and the hole is of such a diameter that the coupling is 40–50 db. For the X-band test coupler presently used, the hole diameter is $\frac{1}{8}$ inch and the wall thickness at the hole is 0.020 inch. The X-band test coupler is illustrated in Fig. 1. The two waveguides are soldered together and an access hole is provided for inserting the sample into the coupling hole. The fit of the cover plate on the access hole is not critical since any leakage of power through it is unimportant in relative power measurements of this type as long as the leakage power remains constant while the measurements are being taken. The ferrite sample is glued symmetrically in the coupling hole with Duco Cement, which had no noticeable effect on the measurements. However, the placement of the sample in the coupling hole had some effect on the level of the coupled power but no noticeable effect on the line width or the *g* factor.

The usual method for measuring the microwave susceptibility and the effective *g* factor of a ferrite depends upon the complex frequency perturbation of a resonant cavity. The method is quite popular but fairly difficult to instrument. It also has other disadvantages:

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† Lockheed Missile Systems Div., Sunnyvale, Calif.

[1] D. C. Stinson, "Ferrite directional couplers with off-center apertures," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-6, pp. 332–333; July, 1958.

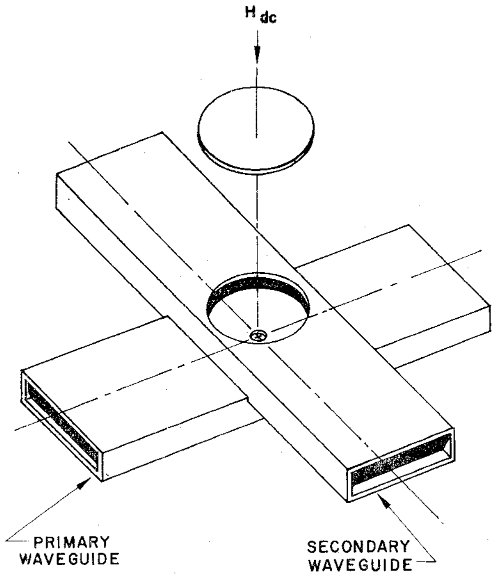

Fig. 1—Cross-guide directional coupler.

1) it requires familiarization with cavity-coupling techniques and Q-measuring techniques; 2) it does not permit one to vary frequency easily; 3) it requires a high-Q cavity, which is often expensive; 4) it requires a rather extensive amount of accessory equipment. Another disadvantage is that the amplitude of the microwave magnetic field in the vicinity of the ferrite is much larger. Thus, it is quite easy to exceed the critical field necessary for the excitation of spin waves.[2] This limitation becomes important when measuring the narrow line widths possessed by many of the garnet-structure ferrites. Regardless of these disadvantages, the cavity-perturbation technique is required in a large number of experiments because of its extreme accuracy and because of its susceptibility to a great many refinements. However, it is felt that the cavity technique can be dispensed with for the routine measurement of the line width and the g factor of microwave ferrites. This is especially true where one is concerned with comparative data among a large number of ferrite samples. In such a situation, saving time is more important than obtaining very accurate data. In view of these considerations, it is felt that the cross-guide coupler method should be extremely useful to anyone who desires information which may be obtained quickly and easily concerning the line width of ferrites, *e.g.*, ferrite manufacturing organizations.

The technique for measuring line widths of ferrites using a cross-guide coupler is easily developed from the theory of coupling through apertures containing ferrites. This theory[3] was formulated recently in fairly general terms and can be used directly. The theory which will require a little more attention is that dealing with the magnetization of the ferrite sample. This is developed fully in the Appendix.

Coupling Theory

The theory which was developed for coupling through an aperture containing a ferrite assumes an aperture which is small compared with the guide wavelength and a ferrite sample which is small compared with the wavelength inside it. Fortunately, experimental data[4,5] indicate that these restrictions are not too severe. Thus, the theoretical results can be quite useful in obtaining qualitative information concerning many ferrite coupling problems. In the present paper, these restrictions create no special problem since only small samples and small apertures are considered.

The coupling expressions derived from the foregoing theory for coupling between waveguides contained terms relating to both the electric and magnetic susceptibilities. However, it was shown that one could remove the electric susceptibility term by terminating the primary waveguide in a short located so as to annul the electric field at the aperture, *i.e.*, a multiple of a half guide wavelength from the aperture. Thus, the coupling expression for a collinear coupler contained only the diagonal magnetic susceptibility while the coupling expression for a cross-guide coupler contained only the nondiagonal magnetic susceptibility. Also, for the cross-guide coupler there is no coupled power in the absence of the applied magnetostatic field. For the collinear coupler, there is coupled power in the absence of the applied magnetostatic field. In view of this fact, the cross-guide coupler is considered more suitable for line width measurements since extremely small samples may be used. In practices, garnet-structure ferrite spheres 0.030 inch in diameter have been quite adequate for measurements at X-band frequencies.[6] Even smaller samples could be used if the sensitivity of the detecting system were increased. The present system uses a conventional waveguide detector mount and a standing wave indicator.

The expression for the coupled power in db in the cross-guide coupler for a centered aperture and with a short located in the primary waveguide so as to annul the electric field at the aperture is

$$C^{\pm} = C_0' + 20 \log |\chi_{xy}| \tag{1}$$

where $C_0' = 20 \log [2\pi d^3(3ab\lambda_g)^{-1}F_H]$; χ_{xy}[7] is the nondiagonal magnetic susceptibility of the ferrite sample in the coupling aperture; d is the diameter of the aperture; a and b are the wide and narrow dimensions, respectively, of the rectangular waveguide; λ_g is the guide wavelength; and F_H is a quantity which measures the attenuation due to finite aperture thickness. The expression (1) and all of the quantities involved are

[2] H. Suhl, "The nonlinear behavior of ferrites at high microwave signal levels," Proc. IRE, vol. 44, pp. 1270–1284; October, 1956.

[3] D. C. Stinson, "Coupling through an aperture containing an anisotropic ferrite," IRE Trans. on Microwave Theory and Techniques, vol. MTT-5, pp. 184–191; July, 1957.

[4] *Ibid.*, Figs. 2, 6, and 7.

[5] Stinson, footnote 1.

[6] The effect on the line width and the g factor of the sample size and the surface roughness of the sample will be reported in a WESCON paper.

[7] The susceptibility used here is defined in terms of the external microwave magnetic field. However, data obtained using external rather than intrinsic quantities are satisfactory as mentioned by E. G. Spencer, L. A. Ault, and R. C. LeCraw, "Intrinsic tensor permeabilities on ferrite rods, spheres, and disks," Proc. IRE, vol. 44, pp. 1311–1317; October, 1956. See p. 1314.

more completely defined elsewhere.[8] MKS units are used throughout. Since relative measured values are of interest, we consider only the magnitude of the magnetic susceptibility, $|\chi_{xy}|$. By measuring the line width of $|\chi_{xy}|$, it is possible to obtain useful comparative information among ferrite samples. However, it would be desirable to measure the true line width which is normally defined in relation to the absorptive component, χ_{xy}', where χ_{xy}' is the real part of χ_{xy}. Thus, since the line width measured using (1) is not the true line width of the material, we need to derive a relation concerning the values of the magnetostatic field and the amplitude of $|\chi_{xy}|$ when χ_{xy}' is half its maximum value. This is carried out in the Appendix. The result is that

$$|\chi_{xy}(H_{3,4})| \; |\chi_{xy}(H_1)|^{-1} = 2^{-1/2}$$

when

$$[\chi_{xy}'(H_{2,5})][\chi_{xy}'(H_0)]^{-1} = \tfrac{1}{2}; \qquad (2)$$

where the expression $\chi_{xy}(H_i)$ means the value of χ_{xy} when the applied magnetostatic field has the value H_i. The relations in (2) are illustrated in Fig. 2. When the results of (2) are applied to (1), the measured line width, $\widetilde{\Delta H}$, and the g factor can be obtained simply by measuring the magnetostatic field values at the two 3-db points and at the peak of the coupled power curve. The g factor and the measured line width are obtained from

$$\widetilde{\Delta H} = H_4 - H_3$$
$$g = 2H_rH_1^{-1}(1 - \tfrac{1}{2}\lambda_1^2)$$
$$\lambda_1 = \widetilde{\Delta H}(2H_1)^{-1} \qquad (3a)$$

where $H_r = \omega[\mu_0\gamma_0'(1+\lambda_1^2)^{1/2}]^{-1}$ and $\mu_0\gamma_0' = 2.210 \cdot 10^5$ met/amp-sec. If one uses CGS units, the expression for H_r in oersteds is $H_r = f(2.8)^{-1}(1+\lambda_1^2)^{-1/2}$, where f is the operating frequency in megacycles. If the reduced damping constant is very small, the expression for the g factor simplifies to

$$g = 2H_{r0}H_1^{-1} \quad \text{for} \quad \lambda_1 \ll 1 \qquad (3b)$$

where

$$H_{r0} = \omega(\mu_0\gamma_0')^{-1}. \qquad (3c)$$

The following measurement procedure for obtaining H_1, H_3, and H_4 has been found satisfactory. The magnetostatic field is first increased until the peak of the coupled power curve is obtained. This peak output is set at an arbitrary reference level by an ordinary variable attenuator and a calibrated variable attenuator in the primary arm. The calibrated attenuator is set at 3 db. The calibrated attenuator is then set at zero db and the magnetostatic field value is decreased considerably below the resonance value. The magnetostatic field is next increased until the coupled power reaches the prescribed reference level. The field value obtained is H_3 in Fig. 2. The magnetostatic field is further increased until the reference level is reached but with the calibrated attenuator set at 3 db. This field value is H_1. The field value H_4 is obtained by setting the calibrated attenuator at zero db again and increasing the magnetostatic field until the reference level is reached. In this method hysteresis effects are avoided by always increasing the magnetostatic field.

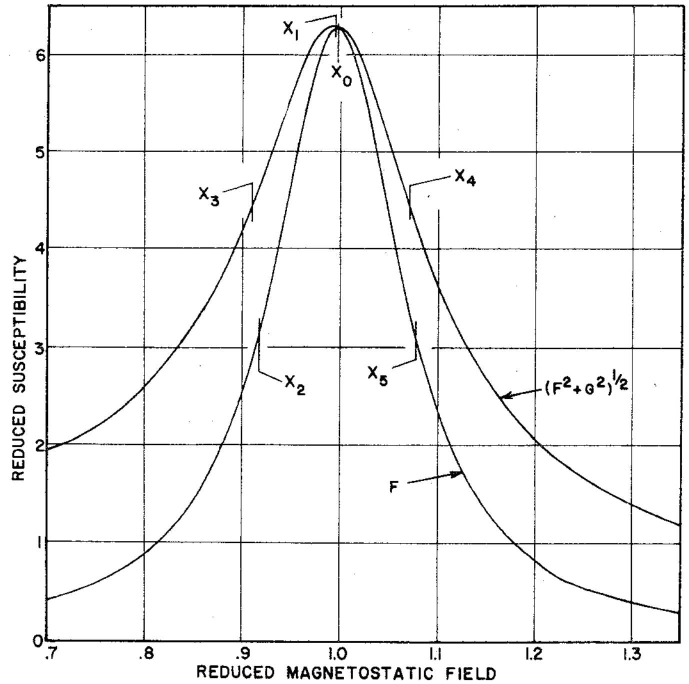

Fig. 2—Relation between the magnitude of the complex nondiagonal susceptibility and the magnitude of its absorptive component as a function of the applied magnetostatic field, all quantities in reduced units.

Experimental Results

The theoretical basis for the foregoing expressions rests upon the usual equation of motion of the magnetization[9] with the Landau-Lifshitz form of damping.[10] Since this theory is adequate for a small-signal analysis, it is felt that measurements of line width and g factor performed with the cross-guide coupler should compare favorably with measurements performed using the cavity technique. The results of such comparative measurements are shown in Table I. The cavity system available for line width measurements was not too refined and consequently accurate to about ±10 per cent, since errors are contained both in the value of the magnetostatic field and the measured Q. The error associated with the coupler measurements was about ±5 per cent, since the 3-db points of the coupled power curves can be measured more accurately than the Q of the cavity. One question which arises is whether there is an error in the measurements because of the nonuniform fields in the coupling aperture. However, it is felt that this problem is not significant since the tangential magnetic fields in the aperture are identical

[8] Stinson, footnote 3, (14), (22), and (28).

[9] C. L. Hogan, "The microwave gyrator," *Bell Sys. Tech. J.*, vol. 31, pp. 1–31; January, 1952.

[10] H. G. Beljers, "Measurements on gyromagnetic resonance of a ferrite using cavity resonators," *Physica*, vol. 14, pp. 629–641; February, 1949.

TABLE I

COMPARISON OF FERRITE PARAMETERS MEASURED IN CAVITY AND CROSS-GUIDE COUPLER

Material	Cavity ΔH oersteds	Coupler ΔH oersteds	Diameter, inches	H_1 oersteds	λ_1	Coupler g factor
General Ceramics R-1	535	473	0.046	3063	0.07725	2.16
Raytheon R-151	450	476	0.040	3150	0.0756	2.10
Lockheed 26-1	60	58	0.030	3326	0.00873	1.99
Lockheed 26-2	60	60	0.030	3324	0.00904	1.99
Lockheed 26-5	60	56	0.030	3326	0.00843	1.99
Lockheed 24A	150	176	0.030	3225	0.0273	2.05
Lockheed 26A	75	76	0.030	3290	0.01155	2.02
Lockheed 26A1	145	169	0.030	3228	0.0262	2.05

with those of the undisturbed system with no ferrite present.[11] Thus, the perturbing effect of the ferrite on the aperture fields can be reduced to a negligible amount by using a ferrite sample considerably smaller than the aperture and by using a very small aperture wall thickness. This affirmation is substantiated by the correspondence between the cavity and coupler line width measurements presented in Table I. Further corroboration is offered in Fig. 3 with theoretical and experimental curves of coupled power vs the magnitude of the applied magnetostatic field. The experimental coupling curve was obtained using the cross-guide coupler with a centered aperture 0.124 inch in diameter, a short located so as to annul the electric field at the aperture, and a LMSD 26C YIG sphere 0.060 inch in diameter. The theoretical curve was obtained by calculating 10 log (F^2+G^2) from (4a) and (4b) in the Appendix and setting the peak value equal to zero db. The value of λ_1 was calculated from (3a) using the measured values of $\widetilde{\Delta H} = 55.5$ oersteds and $H_1 = 3313$ oersteds. The value of g was calculated from (3b) using a value of $H_{r0} = 3315$ obtained from (3c). With an operating frequency of 9272.3 mc, the calculated values were $g = 2.0012$ and $\lambda_1 = 0.0084$. The error between the theoretical and experimental curves is less than 1 per cent, which is better than the accuracy of the magnet calibration.

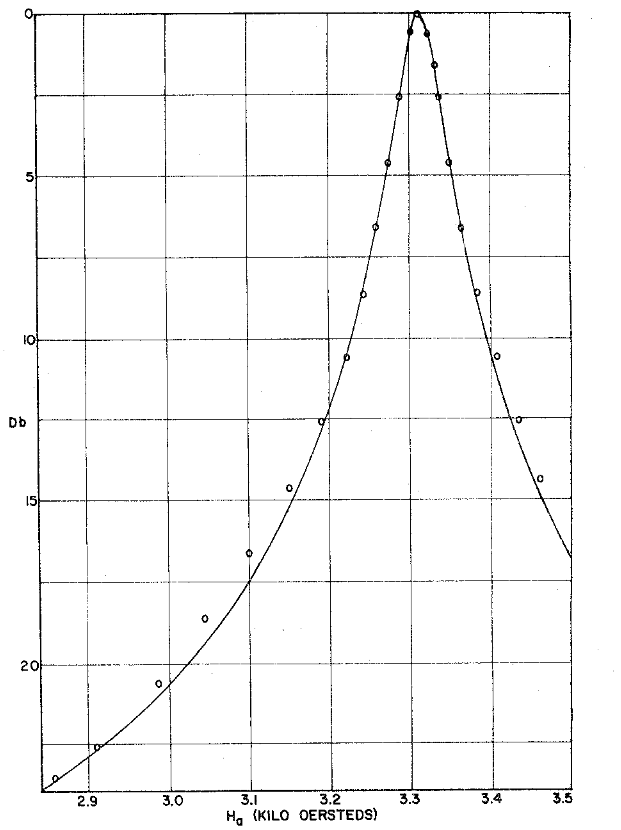

Fig. 3—Comparison of theory and experiment for the coupled power in a cross-guide coupler as a function of the applied magnetostatic field. Theoretical curve calculated for $\lambda_1 = 0.0084$—based upon measured values of $\widetilde{\Delta H} = 55.5$ oersteds and $H_1 = 3313$ oersteds. Experiment performed at 9272 mc.

APPENDIX

MICROWAVE MAGNETIC SUSCEPTIBILITIES

The expressions for the magnetic susceptibilities of a ferrite sphere as a function of the applied magnetostatic field and with the saturation magnetization, damping constant, and g factor as parameters has been derived elsewhere.[12]

The quantities of interest are the following:

$$\chi_{xy}' = EF$$
$$\chi_{xy}'' = EG \tag{4a}$$

where

$$\chi_{xy} = \chi_{xy}' + j\chi_{xy}''$$
$$E = \gamma'\mu_0 M\omega^{-1}$$
$$F = c/b$$
$$G = -c'/b$$
$$c = 2\lambda_1 x(1+\lambda_1^2)^{-1/2}$$
$$c' = 1 - x^2$$
$$b = x^4 - 2x^2[1 - 2\lambda_1^2(1+\lambda_1^2)^{-1}] + 1$$
$$x = \gamma'\mu_0 H\omega^{-1}(1+\lambda_1^2)^{1/2}. \tag{4b}$$

[11] C. J. Bouwkamp, "Diffraction theory," *Rep. Prog. Phys.*, vol. 17, pp. 32–100; 1954. See p. 78.

[12] Stinson, footnote 3, see Appendix.

In the above expressions, γ' is the magnitude of the magnetomechanical ratio, μ_0 is the magnetic inductive capacity of free space, M is the saturation magnetization, $\lambda_1 = \lambda/M$ is a reduced damping constant, and H is the applied magnetostatic field. The problem is that of determining the values of x for which χ_{xy}' is maximum and one half of its maximum value and the values of x for which $|\chi_{xy}|$ is maximum and $2^{-1/2}$ of its maximum value. Although this is a rather involved analytical procedure using the above expressions, a graphical solution can be obtained quite easily. In the present case, this was accomplished by calculating the values of F and $(F^2+G^2)^{1/2}$ as a function of x with a computer for values of λ_1 of 0.125, 0.100, 0.080, and 0.070. The results were then plotted as a function of x and the aforementioned values of x were obtained from the curves, one of which is shown in Fig. 2 for $\lambda_1 = 0.080$. The following values were obtained for x in this manner:

$$x_0 = 1 - \lambda_1^2/2, \qquad F_{\max} = F_C \;@\; x = x_0$$

$$\left.\begin{array}{l} x_2 = 1 - \lambda_1 - \lambda_1^2/2, \\ x_5 = 1 + \lambda_1 - 3\lambda_1^2/5, \end{array}\right. F_0F_{2,5}^{-1} = 2 \;@\; x = x_2,\, x = x_5 \quad (5a)$$

$$x_1 = 1 - \lambda_1^2 \qquad (F^2 + G^2)_{\max} = A_1 \;@\; x = x_1$$

$$x_3 = 1 - \lambda_1 - \lambda_1^2$$

$$x_4 = 1 + \lambda_1 - 3\lambda_1^2/2, \;\; A_1A_{3,4}^{-1} = 2 \;@\; x = x_3,\, x = x_4 \quad (5b)$$

where the x_i's are defined in Fig. 2. The true line width in terms of x is given as

$$\Delta x = x_5 - x_2 = 2\lambda_1 - \lambda_1^2/10.$$

The measured line width in the coupler in terms of x is

$$\widetilde{\Delta x} = x_4 - x_3 = 2\lambda_1 - \tfrac{1}{2}\lambda_1^2. \qquad (6)$$

Thus, if one measures in the coupler the line width $\widetilde{\Delta x}$, the value obtained is 2 per cent larger than the true line width Δx, if the reduced damping constant is 0.10. As the reduced damping constant decreases below 0.10, the measured line width approaches the true line width even more closely. The actual expression for the reduced damping constant can be obtained from (5b), (6), and (4b) as

$$\widetilde{\Delta H}H_1^{-1} = \tfrac{1}{2}(4\lambda_1 - \lambda_1^2)(1 - \lambda_1^2)^{-1}$$

where $\widetilde{\Delta H} = H_4 - H_3$. This expression reduces to the following for λ_1 small:

$$\lambda_1 = \tfrac{1}{2}\widetilde{\Delta H}/H_1. \qquad (3a)$$

The expression for the g factor can be obtained from (5a), (5b), and (4b) as:

$$g = 2H_rH_1^{-1}(1 - \lambda_1^2)(1 - \tfrac{1}{2}\lambda_1^2)^{-1} \simeq 2H_rH_1^{-1}(1 - \tfrac{1}{2}\lambda_1^2).$$

Acknowledgment

The author wishes to thank L. Freiberg and M. Fisher for performing the cavity measurements and J. Novak for performing the coupler measurements and grinding the samples. Thanks are also extended to the Magnetic Properties Group of the Solid-State Electronics Department for supplying the YIG samples.

Applications of Directional Filters for Multiplexing Systems*

FRANKLIN S. COALE†

Summary—**The design of microwave multiplexing systems for frequency channelization of a broad-band microwave spectrum is complicated by problems such as off-resonance mismatch and mutual interaction between adjacent filters. By employing directional filters as basic building blocks, it is possible to construct multiplexing filters with a perfect input match since the input VSWR of a directional filter is theoretically unity both at resonance and off-resonance. Less insertion loss of a manifold may be obtained by the use of directional filters than with conventional band-pass filters. Curves giving the predicted response of a manifold containing *n* elements are presented for single-tuned and double-tuned directional filters. An asymmetrical response shape is obtained which has a midband insertion loss related to the separation of adjacent channels.**

An experimental model consisting of a five-channel multiplexer has been constructed utilizing double-tuned-circular-waveguide directional filters.

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† Microwave Engineering Labs., Palo Alto, Calif.

Introduction

Microwave filters are used extensively in separating or combining signals of different frequencies. It is desirable in some applications to divide the frequency band into many narrow channels which are spaced in such a way that complete coverage is obtained; *i.e.*, any signal within the frequency band will be within the pass band of one or more filters. Most multiplexers of this variety have consisted of a number of filters loosely coupled to a transmission